

A SIMPLE MATHEMATICAL MODEL OF A SINGLE SCULLING TECHNIQUE

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The results in sport in rowing depend on the two most important factors, such as the athlete physical features and the techniques of motion. The assessment and optimisation of rowing techniques are possible only when one disposes the reliable mathematical model predicting the results of the regatta that is the time of covering an assumed distance. A single scull participating in the 2000 meters distance regatta is our subject. The purpose of this study is to create a simplified mathematical model to simulate the rowing boat dynamics. The boat-rower system is treated as a material point here. The oar has a prescribed angular motion vs. oarlock depending on the time. The hydrodynamics force distribution to be developed on the oar's blade has been modelled here. Then, the boat motion was described by a single nonlinear ordinary differential equation (NODE). The proposed simple model gives the possibilities of fast and reliable simulation of the single sculling technique and forecasts the result of rowing regattas.

KEY WORDS: rowing, mathematical model, motion's equation, hydrodynamic force on the oar

INTRODUCTION: A single sculling is rowing with two oars, each in one hand. This discipline involves passing a specific distance in the shortest possible time and therefore the efficient sculling technique is of a paramount importance. This paper attempts to develop the mathematical model of rowing to be useful for analysis, learning, improvement and optimization of motion techniques. According to Zatsiorsky and Yakunin (1991) "Despite relatively numerous studies, the biomechanics of rowing remains poorly understood". These authors were among the first ones who attempted the mathematical description of rowing. A lot of authors have dealt with similar subjects, among others. Pulman, C. (access 2014), gives a thorough description of the phenomena concerning different aspects of rowing. Some authors, such as Findlay, M. and Turno S.R. (2010), Kinoshita et al. (2008), and Millar S.K. et al. (2015) have treated the similar problems, especially the hydrodynamics phenomena related to flow past the oar. The main aim of the present work is to build a mathematical model allowing to predict a sports result of the single scull regatta.

METHODS: The mathematical model of rowing technique has been derived from the second Newton's dynamics law and rules describing hydrodynamic forces, for the physical model of rowing shown in Figure 1. There are the equation of motion of the rower-boat system with its elements to be shown below:

$$m\ddot{x}(t) = T_{oa}(t) - k_b \dot{x}^2(t)$$

Where propulsion force is: $T_{oa}(t) = 2F(t) \sin \gamma(t)$

where the hydrodynamic force $F(t)$ is as follow:

$$F(t) = 0.5\rho_{H_2O} \int_{l_{OAmin}}^{l_{OAmax}} \left[\frac{d\gamma(t)}{dt} l_{OA} - \frac{dx(t)}{dt} \sin \gamma(t) \right]^2 b_{OA} dl_{OA} c_{DOA} \left\{ \frac{1 + \operatorname{sgn} \left[\frac{d\gamma(t)}{dt} \right]}{2} \right\} \left[\frac{1 + \operatorname{sgn} g(t)}{2} \right]$$

with the adjusting term:

$$\operatorname{sgn} g(t) = \operatorname{sgn}[0.5(l_{OAmax} + l_{OAmin})\dot{\gamma}(t) - \dot{x}(t) \sin \gamma(t)]$$

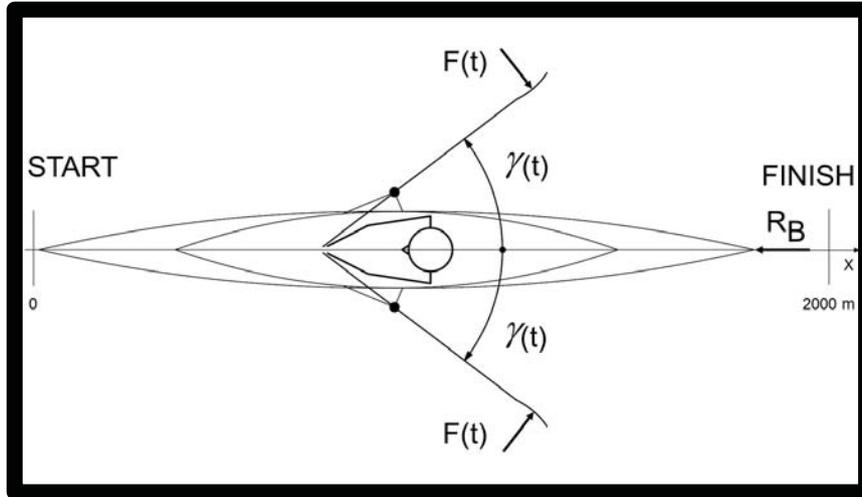


Figure 1: The physical model of rowing, $\gamma(t)$ – an oar angle, $F(t)$ – a force developed by the oar, $R_B = k_B(v_B)^2$ – a hydrodynamics drag of the boat (v_B – boat velocity).

The angle γ of oar vs. time t is defined as the function:

$$\gamma(t) = 0.5(\gamma_{max} + \gamma_{min}) - 0.5(\gamma_{max} - \gamma_{min}) \left\{ k_1(t) \cos \left[\frac{2\pi(t - iT)}{T_1} \right] - k_2(t) \cos \left[\frac{2\pi(t - 0.5T_1 - iT)}{T_2} \right] \right\} \quad \forall_{0 \leq t \leq \infty}$$

$$T = 0.5(T_1 + T_2) \quad (T_1 > T_2) \vee (T_1 < T_2) \vee (T_1 = T_2 = 0.5T)$$

$$\left. \begin{aligned} (k_1(t) = 1) \wedge (k_2(t) = 0) &\Leftrightarrow iT \leq t < iT + 0.5T_1 \\ (k_1(t) = 0) \wedge (k_2(t) = 1) &\Leftrightarrow iT + 0.5T_1 \leq t < (i+1)T \end{aligned} \right\} \quad \forall_{0 \leq t \leq \infty}$$

The oar's angular velocity formula is shown below:

$$\frac{d\gamma(t)}{dt} = \pi(\gamma_{max} - \gamma_{min}) \left\{ \frac{k_1(t)}{T_1} \sin \left[\frac{2\pi(t - iT)}{T_1} \right] - \frac{k_2(t)}{T_2} \sin \left[\frac{2\pi(t - 0.5T_1 - iT)}{T_2} \right] \right\} \quad \forall_{0 \leq t \leq \infty}$$

The solution of the equation of a motion for different parameters is a tool of simulation.

RESULTS: The following values given below are listed parameters: the system total mass (rower, boat, oars) $m=100$ kg, a minimal angle of an oar stroke $\gamma_{min}=30^\circ$, a maximal angle of an oar stroke $\gamma_{max}=120^\circ$, a drag coefficient of the boat $k_B=3.8$ kg/m, a dimensionless drag coefficient of the oar $c_{DOA}=1.13$, water density $\rho_{H_2O}=1000$ kg/m³, the distance between an oar lock and the oar blade tip $l_{OAmax}=2.04$ m, the distance between oar lock and the oar blade root $l_{OAmin}=1.6$ m, the oar blade width $b_{OA}=0.23$ m, the total rowing cycle period $T=2$ s, the active phase of a cycle period $T_A=0.5T_1=0.7$ s, the passive phase of cycle period $T_B=0.5T_2=1.3$ s. The chosen examples of the computer simulation results of rowing at the distance of 2,000m are shown in the Figures no 2 - 3. These Figures show the motion parameters during the first ten seconds from the start of regatta. The system is modeled using MATLAB[®] package. The resulting equation of motion is solved with ode45 code (Shampine and Reichelt, 1997) that is based on the explicit Runge-Kutta formula of orders 4 and 5. The equation is solved with relatively tight error tolerances of 10^{-10} for absolute and 10^{-8} for relative tolerances. The use of the default tolerance values results in a wrong solution. For the given model, the boat travels 2000m after approximately 7 minutes and 55 seconds (475 s). On replacing the limit angles $\gamma_{min}=30^\circ$ and $\gamma_{max}=120^\circ$ by $\gamma_{min}=45^\circ$ and $\gamma_{max}=135^\circ$ the simulation of 2000 meters regatta showed the resulting time of 8 minutes and 1s (481 s). The main criterion for the

evaluation of the model was the time to overcome the distance of 2000m obtained by simulation on the basis of the assumed frequency, active and passive phase of rowing. The maximum relative error of the race result prediction did not exceed 10%.

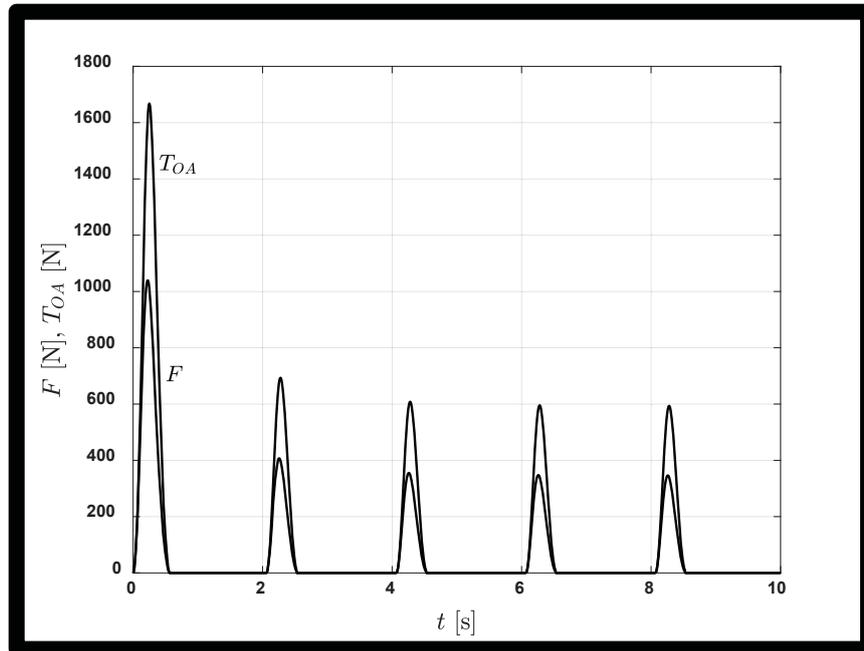


Figure 2: The force developed by the oar F and the resulting propulsion force T_{OA} .

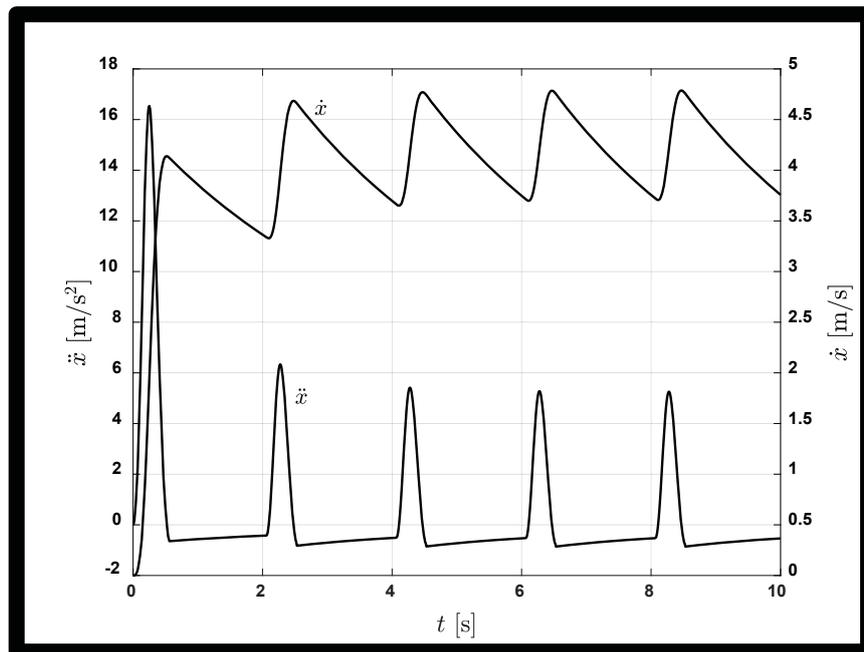


Figure 3: The boat acceleration and the velocity during the first 10 seconds.

DISCUSSION: The following authors: Day et al. J Formaggia, L. et al. (2008), Formaggia, L. et al. (2009) dealt with similar issues by paying their attention to the formation of the hydrodynamic forces phenomenon on the oar. The results of these studies describe the course of the forces on the oar blade according to the experimental data very well. On the other hand, as a result of our assumptions about the cosinusoidal course of an oar angle, we have obtained an essentially different shape of the force variation on the blade. The obtaining the solutions of that NODE for different parameters to have occurred here gave the

possibility to simulate a rowing technique on a single scull and to predict the influence of the chosen parameters on a regatta result. In the literature, the lack of reports concerning the mathematical models allowing the single scull rowing simulation does not permit to study the influence of the rowing technique elements on the outcome of regatta. The usefulness of this sculling model to analyze the impact of the movement technique on the outcome of the race over a distance of 2000m provides the simulation results of the regatta for the range of angle paddling 90° once for the limit values of the paddle angles, such as $\gamma_{\min}=30^\circ$ and $\gamma_{\max}=120^\circ$ and the other time with $\gamma_{\min}=45^\circ$ and $\gamma_{\max}=135^\circ$. The race result for the angles $\gamma_{\min}=45^\circ$ and $\gamma_{\max}=135^\circ$ was about 6 seconds worse than for the previous angle limits. The simulation results match our experimental data and references of other authors. Such a model seems to be able to allow the sculling technique optimization.

CONCLUSION: i) This model provides the opportunity to study the effect of different rowing technique elements on the outcome of the regatta. ii) The model must be verified with a particular care to the phenomenon of power rowing, depending on the oar angular velocity. iii) The model must be expanded to 4 degrees of freedom including: the location related to a regatta course, a boat pitch angle, the trunk tilting and the sliding seat position.

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