ON THE UNCERTAINTY OF MEASUREMENTS CONCERNING THE CENTER OF PRESSURE SIGNALS

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Force plate measurement systems unavoidably introduce noise into the output signals. Noise in the center of pressure (COP) signal is the propagation of error that arises from the combination of the components used to compute it. A framework to analyze the random error in COP signals is introduced based on the “Guide to the Expression of Uncertainty of Measurement” (GUM) approach.

KEYWORDS: Digital Filtering, Smoothing, Uncertainty of Measurement, Center of Pressure, Variance Matrix.

INTRODUCTION: Force plate measurement systems unavoidably introduce noise into the output signals (Hunt, 1998). The noise in the COP signal is the propagation error that arises from the combination of the components used to compute it (i.e. strain gauges and/or transductor) with function f. Generally, the noise is modeled as a wide-band additive, stationary, zero-mean, and uncorrelated noise that contaminates the low-pass COP signal with noise variance \( \sigma^2 \). However, even if the noise of the recorded GRF signals can be modeled as an additive zero-mean “white noise”, the nonlinear transformation in COP computation induces noise in the COP signal that becomes nonstationary (i.e., unequal noise variance), except for the case where the \( F_z \) (vertical force component) is constant. In many fields of biomechanical studies, there are instants where \( F_z \) vector of GRF changes its magnitude drastically during its evolution in time, so, the noise presented in the raw COP coordinates should be modeled as additive, zero-mean, and nonstationary – i.e., unequal noise variance across COP coordinates and variations in time of the noise power. Weak correlated noise is also assumed. This renders time-invariant Fourier transform based filtering techniques or Generalized cross-validated splines (GCVSPL) as suboptimal. In dynamics tasks, many ways exist to process force plate measured signals. Therefore, the aim of this study was to analyze the noise in the COP signal under different experimental conditions to construct a weighted variance matrix for the COP noise in conformance with the GUM approach (BIPM, IFCC, IUPAC, & ISO, 2008).

METHODS: General Uncertainty Framework. Consider a single real output quantity Y that is related to a vector of real input quantities \( X = (X_1, \ldots, X_N) \) T by an explicit univariate measurement model \( Y = f(X) \). The estimate of the output quantity is \( y = f(x) \) with \( x = (x_1, \ldots, x_N) \). Assuming linear or weakly-nonlinear relation and using a first-order Taylor series expansion, the standard uncertainty \( u_y \) associated with \( y \) is obtained by the “law of propagation of uncertainties” expressed by

\[
    u_y^2 = S_x U_x S_x^T
\]

where \( S_x \) is the vector of the sensitivity coefficients expressed by the values of the partial derivatives \( \frac{\partial f}{\partial X_j} \) for \( j = 1, \ldots, N \), at \( X = x \) and \( U_x \) is the \( N \times N \) uncertainty matrix associated with \( x \) containing the covariances \( u(x_i, x_j) \) for \( i, j = 1, \ldots, N \) associated with \( x_i \) and \( x_j \).

Experimental Setup. Two strain-gauge force platforms (Dinascan 600M, IBV, Valencia, Spain) were utilized to obtain the temporal evaluation of the components of the GRF vector and the coordinates of the COP during the experiment at a sampling rate of 30 Hz. Calibrated dead loads (Telju, Spain) were used.
The force measurement system was switched on always 15 min prior to the measurement process to reach thermal stability. A calibrated dead load (M) was displaced on the top plate of the force platform from the point P₀ to the point P₁₀ gradually in 10 consecutive stages that corresponded to 11 fixed points and then was returned back from the point P₁₀ to the point P₀ in a similar manner. This operation was repeated two times. During the displacements of the dead load M from the point P₀ to the point P₁₀ and back to the P₀, at each one of the intermediate points where the load M was placed gradually, the COP was measured for 10 repeated times in an interval of 30 sec between each repetition. Each repeated COP data was collected for a time period of 5 sec at a sampling rate of 30 Hz under a set of repeatable conditions of measurement. To ensure that the load M was placed with accuracy onto the fixed points on the grid, a point loader was used. This procedure was repeated for different calibrated loads (range: from ≈ 98 N to ≈ 294 N) for the two force platforms used in the study. In addition, a dead load (≈ 294N) was placed on the top plate of the force platform about its geometrical center and the COP was registered at 30, 230 and 500 Hz for 10 sec. A frequency analysis was made on the COP signals to test whether COP noise is “white”. To test the influence of the sampling rate another dead load (≈ 294N) was placed on the top plate of the force platform about its geometrical center and the COP was registered for 10 sec at an integer sequence of frequencies (30, 40, 50, ..., 300 Hz).

Data Processing and Analysis. There are k = 10 repeated COP samples comprised of 150 data each, for each one of the 11 fixed points P₀→₁₀, replicated r = 4 times. For each point, the mean value and standard deviation for each repeated COP sample were computed, as well as the overall mean value comprised of all the data of the k = 10 repeated samples. The overall mean (\(\overline{y}\)) and standard deviation (\(S_1\)) were computed by:

\[
\overline{y}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} y_{ij} \quad \text{and} \quad s_i = \sqrt{\frac{1}{n_i-1} \sum_{j=1}^{n_i} (y_{ij} - \overline{y}_i)^2} \quad \text{with } i = 1, \ldots, k.
\]

<table>
<thead>
<tr>
<th>(n_i) the (i)th repeated COP sample size</th>
<th>(\overline{y}_i) mean value for the (i)th repeated COP sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_i) standard deviation of (i)th repeated COP sample</td>
<td>(y_{ij}) the (j) datum of the (i)th repeated COP sample</td>
</tr>
</tbody>
</table>

The overall mean value for the \(k = 10\) repeated samples at each point was computed as:

\[
\overline{\overline{y}} = \frac{1}{n} \sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij} = \frac{1}{n} \sum_{i=1}^{k} n_i \overline{y}_i
\]

where \(n = \sum_{i=1}^{k} n_i =\) total number of measurements. In total, four overall means were calculated at each point, one for every replication. The standard deviation, \(s\), of all repeated COP samples for one replication for each point is (NASA, 2010).

\[
s = \sqrt{s_b^2 + s_w^2}
\]

The standard deviation of the sampled mean values relative to the overall mean value is the between sample sigma, \(S_b\), computed as and the standard deviation within samples is the within sample sigma (noise), \(S_w\), computed as

\[
s_b = \sqrt{\frac{1}{n-1} \sum_{i=1}^{k} n_i (\overline{y}_i - \overline{\overline{y}})^2} \quad \text{and} \quad s_w = \sqrt{\frac{1}{n-1} \sum_{i=1}^{k} (n_i - 1) s_i^2}
\]

RESULTS AND DISCUSSION: The results showed that for \(F_z = \) constant the noise of the COP signal could be modeled as additive, zero-mean “white noise” (Fig. 1 and 2). Different sampling rates influence the COP noise. This is obvious for the COP signals that were registered with a
very different (low - very high) sampling rate (Fig. 1). For example, the variances of the raw COP data for different sampling rates are RAWy-30Hz = 0.54 mm², RAWy-230Hz = 0.59 mm², RAWy-500Hz = 0.61 mm², and RAWx-30Hz = 1.10 mm², RAWx-230Hz = 1.20 mm², RAWx-500Hz = 1.30 mm² (Fig. 2). However, for a narrower frequency interval, the assumption that the sampling rate does not influence the COP noise can be considered as correct. Notwithstanding, the noise elimination, was higher after oversampling spread the power over higher frequencies. The variance of the COP signals after low-pass filtering was BTWx30Hz = 0.24 mm², BTWx-230Hz = 0.05 mm², BTWx-500Hz = 0.03 mm² and BTWy-30Hz = 0.15 mm², BTWy-230Hz = 0.03 mm², BTWy-500Hz = 0.02 mm² (Fig. 2). Other studies have also shown that cut-off frequency and sampling rate influence stabilometric parameters (Scoppa, Capra, Gallamini, & Shiffer, 2013; Schmid, Conforto, Camomilla, Cappozzo, & D’Alessio, 2002). However, our study demonstrated that cut-off frequency and sampling rate are also dependent on the magnitude of the vertical force.

Figure 1. Power spectral density of the raw COP signals obtained at different sampling rates (only the COPx is shown).

Table I. Standard uncertainty $F_z (n)$ obtained for different dead load weights.

<table>
<thead>
<tr>
<th>Force Platform</th>
<th>M₁₀</th>
<th>M₂₀</th>
<th>M₃₀</th>
<th>M₄₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force Platform 1</td>
<td>1.90</td>
<td>1.86</td>
<td>2.00</td>
<td>1.84</td>
</tr>
<tr>
<td>Force Platform 2</td>
<td>1.96</td>
<td>1.81</td>
<td>1.75</td>
<td>2.08</td>
</tr>
</tbody>
</table>

Figure 2. COP signals with different sampling rates (30, 230 and 500 Hz) after low-pass filtering with a fourth-order zero- phase- shift Butterworth filter (BTW) with cut-off frequency at 5 Hz (Winter, 2009). The probability function for the distribution is shown for each time series (only the COPx is shown).
There was not a trend among the standard uncertainty of the $F_z$ signal registered with the different dead load weights (Table I). Therefore, the highest standard uncertainty of both force platforms, ($u_{F_z} = 2.1$ N) was chosen (Table I). The $u_y$ in the COP measurements was modeled as a hyperbolic function of the $F_z$ magnitude ($u_{F_z} = c$). According to Lees and Lake (2008) the minimum $u_y$ is obtained when the dead load is placed at the geometrical center of the top plate of the force platform as in this point the same fraction of the $F_z$ is registered by each load cell. Fig. (3) shows the experimentally obtained noise curve together with the minimal curve of the model for two cases of statistically correlated error sources, $\rho = -0.9$ and $\rho = -0.8$. The variance explained by the fitted regression models ($R^2$) are very high and their match with the error model is obvious. For the Y-axis the experimentally obtained COP uncertainty is better modeled with statistically correlated error $\rho = -0.8$, while for the X-axis with $\rho = -0.9$.

![Graph showing the noise curve](image)

**Fig. 3. Values of the $u_y$ obtained by the “propagation law”, along with the experimentally obtained standard deviation data fitted with regression lines.**

The colored areas are bounded by the values of $u_y$ for $\rho = -0.9$ (low) and $\rho = -0.8$ (up). Blue and red areas correspond to the $u_y$ of the X and Y axes, respectively (F1X= Force platform 1 X-axis; F1Y= Force platform 1 Y-axis; F2X= Force platform 2 X-axis; F2Y= Force platform 2 Y-axis;)

**CONCLUSION:** The implementation of the GUM approach (BIPM et al., 2008) to calculate standard uncertainties for specifying the weighted factor for each coordinate of the noisy COP data was introduced. The noise curves can be used (the experimental or the theoretical) in order to obtain the weighted matrix for smoothing purposes. Studies have to take into consideration how acquisition settings like sampling rate, cut-off frequency, and $F_z$ magnitude influence the COP values.

**REFERENCES:**


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