EFFECTIVE USE OF ANGULAR MOMENTUM FOR ROTATIONS ABOUT THE LONGITUDINAL AXIS - EXAMPLES OF QUADRUPLE JUMPS IN FIGURE SKATING

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Quadruple jumps (QJ) have conquered the competitions in figure skating. In men's competitions three or four different QJ is standard for top skaters. The key task of the study with 17 excellently executed QJ was to investigate the influence of the angle between the principal axis of inertia (PAI) and the vector of the angular momentum (AM) on the effective use of the angular momentum for a high angular velocity in the flight. The smallest possible angle between PAI and AM results in the most effective use of the angular momentum and a stable axis of rotation. It is of great interest for sports practice to find out which options can compensate larger angles between PAI and AM. The study shows that QJ can excellently performed even with an angle of 20 to 30 degrees between the PAI and AM. In this case, a larger angular momentum and a longer flight time are required.

KEY WORDS: biomechanical analysis, figure skating, jumping, kinematics.

INTRODUCTION: Since the 2018 Winter Olympics, the medals in the men's competition have been awarded to figure skaters who presented more than two different QJs with an excellent landing (Weigmann-Faßbender & Knoll, 2019). Successful jumps are characterised by complete rotations. If the rotation is incomplete the QJ will be downgraded or it ends in a fall. Falls can happen due to various causes. The flight time can be too short, the rotation incomplete, or the body's longitudinal axis tilts too much. The flight time in QJ is greater than in triple jumps (King, Smith, Higginson, Muncasy and Scheirman, 2004). Different absolute values of angular momentum (AM) at take-off could be determined for QJ with the same flight times (Knoll & Härtel, 2005). The moment of inertia with respect to the longitudinal axis is also relevant in the sense of a necessary condition. At the last contact on ice and in flight, the moment of inertia (MOI) should be as small as possible. The skater should keep his closest position as long as possible (Knoll & Seidel, 2015).
In this study we investigate the angle between PAI and AM. Effective rotation can be achieved if PAI and AM match. The larger the angle between AM and PAI, the lower the angular velocity for constant AM (Knoll & Hildebrand, 1996). In that study computer simulation indicated the highest angular velocity if the angle between PAI and AM during the flight is zero. But in reality QJ are also successful with angles between 20 and 30 degrees. Figure 1 shows the PAI and AM at take-off of a real quadruple Toeloop (4T) and a real quadruple Salchow (4S).

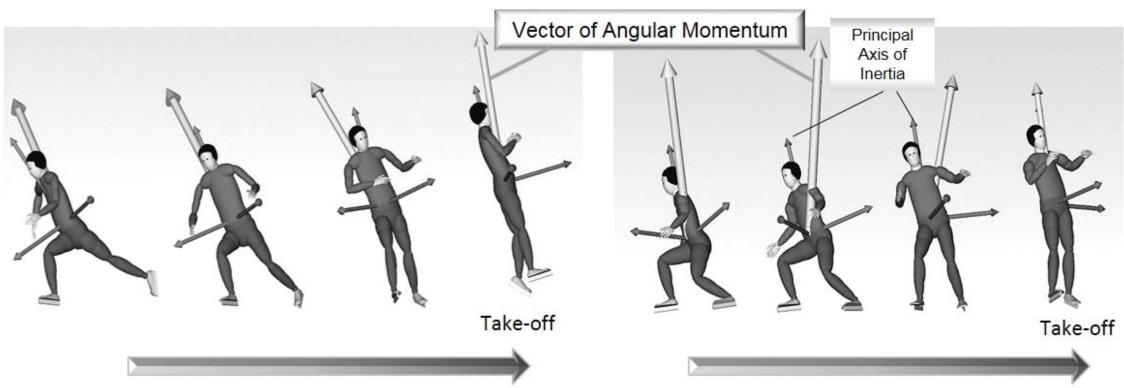

Figure 1: Schematic view of vector of angular momentum (AM) and principal axis of inertia (PAI). Left: take-off for a quadruple Toeloop. Right: take-off for a quadruple Salchow.

Computer simulation shows that an angle of 30 degrees between PAI and AM causes a loss of angular velocity by 3.6 rad/s and a loss of rotation by 0.4 revolutions. This loss needs to be compensated for QJ. It arises the research question how to compensate this loss.

METHODS: We selected best solutions with clean landing of quadruple Toeloop (4T) and the quadruple Salchow (4S) performed by male skaters between 2000 and 2017 in the following competitions: Grand Prix in Gelsenkirchen, World Championships 2004 in Dortmund, Junior World Championships 2007, Nebelhorn Trophy 2012, 2015 in Oberstdorf and national events. "A clean landing was defined as a jump for which the skater landed on one foot and glided out the landing without touching down a hand or having to step out of the landing onto the other foot." (King et al, 2004, p. 111). In this study 17 QJ of 14 male skaters at an age of 16 to 26 years were analysed. The 3D-mesurement systems we used have been described in Knoll & Härtel (2005) and in Knoll (2018).The rotational velocity (RotVel) of the shoulder and hip axis as well as the blade of the landing leg was determined by the assignment of three body points (Figure 2).

The angle between AM and PAI is used to divide the 17 QJ into three groups G1, G2, and G3. Here G1 (n = 4) denotes the group with an angle from 0 to 10 degrees. G2 (n = 6) is the group with an angle between 10 degrees and 20 degrees and G3 (n = 7) is the group with an angle greater than 20 degrees. In the simulation study, the angular velocities to the angles 0, 10, 20 and 30 degrees were calculated (Knoll & Hildebrand, 1996).

Figure 2: Assignment of the hip, the shoulder and blade points for determining of the rotational velocity.

The statistical analyses examined the differences between the mean values in the three groups with regard to relevant parameters. A t-test of two groups each is carried out.

The alaska-viewer 9.5.0 (Institute of Mechatronics, Chemnitz, Germany, 2015) was used to visualize the vector of angular momentum and PAI.

To analyse the rotation efficiency in flight, the 3D data of the world coordinate system were transformed into the athlete's internal coordinate system. The AM is constant during free flight conditions. The body position of the skater is described frame by frame by the three principal axes of inertia (Hildebrand, 1997). The center of mass is the common origin of vector AM as well as of the athlete's coordinate system.

Figure 3 illustrates the three projections of the AM vector L onto the mean axes: L_{eff} corresponds to longitudinal axis, L_x corresponds to transversal axis and L_y corresponds to sagittal axis. Precession movement arises if the directions of L and the longitudinal axis differ from each other.

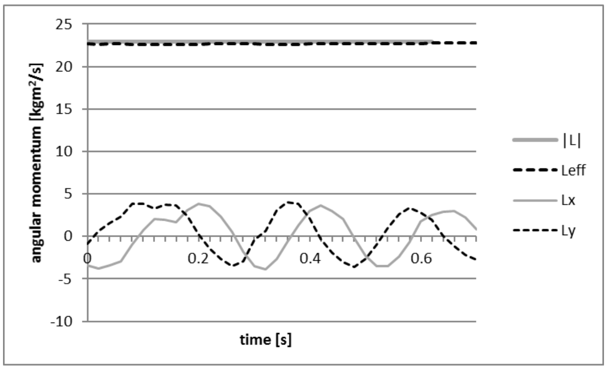

Figure 3: Total AM (|L|), the effective AM (L_{eff}), the part of AM in x direction (L_x), the part of AM in direction y (L_y). One example of a QJ where the transversal and sagittal parts of AM decrease.

RESULTS: The aim of the study is the effective use of the AM in the flight of QJ and to find out which strategies are used to compensate an angle between PAI and AM of more than 20 degrees in the QJ of group 3 (G3). The average RotVel is almost identical in all three groups. The differences in the mean rotational velocity (RotVel) in flight are not significant. The mean value difference of the total AM at the take-off between group G1 and group G3 is significant (r_{AM} = 0.003; p = 0.05). The mean value difference of the total AM between group G2 and group G3 is also significant (r_{AM} = 0.005; p = 0.05).

The jumps of group G3, have larger mean flight times (Table 1). The mean values of the flight time between group G1 and group G3 has only a low significance (r_{tFL} = 0.04; p = 0.05).
A larger flight time is not caused by higher vertical velocity, but by a steeper take-off angle (Table 1). Statistical significance of the differences in these mean values could not be proven. The other comparative values generally differ only slightly and only indicate trends.
All jumps achieve high average rotational velocities (RotVel) of the hips, but different overall rotations are achieved (Table 1).

Table 1: Parameters for quadruple jumps. Values for certain events (Mean ± SD).

	G1 (0° to 10°)	G2 (10° to 20°)	G3 (over 20°)
Angle PAI/AM [°]	5.1 ± 3.6	16.1 ± 2.9	26.7 ± 2.6
Flight time [s]	0.68 ± 0.04	0.71 ± 0.05	0.73 ± 0.03
AM $L_{take-off}$ [kgm²/s]	23.6 ± 11.1	24.3 ± 3.1	32.1 ± 5.5
AM $L_{maximum}$ [kgm²/s]	25 ± 4.2	35.4 ± 4.9	43.6 ± 7.8
Take-off angle [°]	41.3 ± 6.3	42.4 ± 3.7	44.6 ± 3.3
Take-off $velocity_{horizontal}$ [m/s]	3.8 ± 0.7	3.6 ± 0.4	3.6 ± 0.4
Take-off $velocity_{vertical}$ [m/s]	3.4 ± 0.2	3.5 ± 0.3	3.5 ± 0.3
RotVel – hip/in the air [°/s]	1575 ± 63	1579 ± 59	1521 ± 36
Rotation of hip [turns]	2.94 ± 0.06	3.07 ± 0.18	3.0 ± 0.1

The angle between PAI and AM may change during the flight as our study showed. Of the 17 jumps we studied, three showed an increase in the transverse parts of the angular momentum, four remained constant and ten reduced the transverse parts of the angular momentum.
The mean value of the MOI in the last 0.20 s of the flight is smaller, than during the entire flight and at the first contact on the ice. The landing is performed with a very high rotational speed at the first contact on the ice (Table 2, RotVel).

Table 2: Parameters in preparation for landing.

Parameters	Mean/SD
RotVel – in the air [°/s]	1521 ± 36
MOI - in the air [kgm²]	1.7 ± 0.28
MOI - take-off [kgm²]	3.4 ± 0.4
RotVel – hip/last 0.20s [°/s]	1620 ± 65.2
MOI - last 0.20s [kgm²]	1.3 ± 0.3
MOI - first ice contact [kgm²]	1.7 ± 0.5
RotVel – hip/first ice contact [°/s]	1225 ± 233
RotVel – shoulder/first ice contact [°/s]	1234 ± 150
RotVel – blades/first ice contact [°/s]	1245 ± 261
Rotation of the hip [turns]	3.0 ± 0.1

To ensure the necessary rotation, some skaters reduce their MOI during the last 0.20 s of the flight in order to increase the RotVel. With the first contact on the ice the MOI increases slightly. The mean value of the RotVel of the shoulder, hip and blades have the same high rotational speed (Table 2).

DISCUSSION: There are probably different strategies available to perform the required number of turns. In all three groups the average RotVel is almost identical, which was expected, because all QJ are successful jumps. One strategy is to generate large AM (both $L_{maximum}$ and $L_{take-off}$). It turned out that large angles between AM and PAI in the jumps of G3 are compensated by a larger absolute value of the AM. The increase of the flight time is a second option for the compensation of the larger angle between PAI and AM. Another strategy is the more effective use of a smaller AM. This third possibility to make the rotation more effective consists in approaching the PAI and vector of the AM during the flight. The smaller the difference between L and L_{eff}, the smaller are the parts of L_x and L_y. But the larger difference

between L an L_{eff} the larger are the parts of L_x and L_y. This makes the use of AM ineffecvtive and the longitudinal axis will tilt more. The approach of PAI and AM in flight can be achieved via movements of the legs and arms and through changes in body position. This correction of the body position can take place immediately after take-off, when the closed flight position is taken.
The majority of the QJs have a slightly inclined longitudinal axis as King et al (2004) already confirmed. For a perfect landing, the landing leg should be glided on the backward outward edge of the blade. The outward edge position of the blade can be reached by a slight inner circle position of the body. Therefore, a body position change to this flight position is advantageous.
Special attention is paid to the landing of QJ, because it determines the successful execution of the jump. Because in QJ the moment of inertia (MOI) should be in all parts of the flight as small as possible, a tilting of the longitudinal axis is dangerous. With an angle of more than 20 degrees to the PAI and AM, the longitudinal axis begins to tilt visibly. The very narrow flight position does not allow corrections of a tilted longitudinal axis for an optimal landing.
We did not expect that the shoulder and hip axes as well as the blades have such a high rotational speed at the first contact on ice. A strong decrease of the RotVel to the first ice contact could not be found. That makes the landing on one skate very difficult.

CONCLUSION: Quadruple jumps are very complex motions. However, for successful jumps, limited areas for compensation options are possible. To identify these limits and successful strategies or solutions of execution this study was very helpful. The effective use of the generated angular momentum, sports technical solutions have to be developed both for the take-off and the flight to keep the angle between AM and PAI as small as possible. The stability can be improved in this way and falls can be prevented. Three options were obtained to compensate for an angle between PAI and AM of more than 20 degrees. The increase of flight time and angular momentum are possible. The achievement of effective AM should be further investigated in detail.

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